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Fairfield Smith: Effect of Fertilisers on Growth of
Hevea: a Study in Combination
of Data from a Heterogeneous
Group of Experiments.

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CORRECTIONS

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|---------------|-------------------|---|
| Page 89, line | 16: | insert "thin" between "much" and "virgin" |
| ,, 96, ,, | 3: | insert "h" after $\sum_i \sum_j$ |
| ,, 97, ,, | 5: | delete bar above x_{ijk} |
| ,, ,, ,, | 6: | add, bar above x_i (in the text). |
| ,, ,, ,, | 21 | insert (before A_j |
| ,, 109, ,, | 11-12: | insert "logarithm of the" between "the" at the end of line 11 and "mean" at the beginning of line 12. |
| ,, 114, | Footnote: | transpose the last line below the footnote to become the last line of the text. |
| ,, 115, | Literature cited— | |
| | | line 5: for 183 read 138 |
| | | line 3 from bottom: for 110(2) read 110(3): 257-9. |
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- | | | |
|-----------------------------|-----|---------------------------------|
| Page 50, line | 1: | insert) after 10. |
| ,, 54, ,, | 25: | for "specific" read "specified" |
| ,, 56, line 10 from bottom: | | insert) after d_w |
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- Page 42, first line below table: for "The chief point to note from the table above is" read "The above table shows"

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Foreword and Acknowledgements

The first draft of this paper was written while the author was guest research consultant at the Institute of Statistics of the University of North Carolina on a project under contract with the Office of Naval Research to study statistical methods of combining data from groups of experiments. Consequently the paper has been written with two objectives (1) to provide illustrative material for statisticians interested in that problem (2) to present and summarise evidence obtained by the Rubber Research Institute of Malaya on the effect of fertilisers on growth of rubber trees. Separate papers devoted to each objective would entail an amount of duplication that seems unwarranted. Therefore I trust that plantation experts will excuse explanation of some points obvious to them.

I have to record my gratitude and indebtedness to the Office of Naval Research, Navy Department, Washington, D.C., for supporting this work; to Prof. W. G. Cochran who was directing the project and gave invaluable assistance; and to officers of the Soils Division of the Rubber Research Institute, who provided the data.

EFFECT OF FERTILISERS ON GROWTH OF HEVEA.

A study in combination of data from a Heterogenous Group of Experiments.

BY

H. FAIRFIELD SMITH

Summary

Twenty three experiments in which fertilisers were applied during the first two and a half to four years from planting, show the following increases on girth, at six to eight years, as due to fertilisers. The quantities of fertilisers quoted are totals per tree applied throughout the period.

	Inland Soils	Alluvial Soils
Lime ..	No response	No response
Nitrogen (6 oz. N) ..	.26 inch	No response
Potash (2 to 11 oz. K ₂ O) ..	No response	— .44 inch
„ (?) (1.34 oz. N + 1.56 oz. K ₂ O in presence of 5.85 oz. P ₂ O ₅) ..	— .42 inch	—
Phosphate (5.3 to 9.6 oz. P ₂ O ₅)	1.69 inch	.18 inch

Responses were equal (within the limits of uncertainty) on both replantings and new plantings. (But the comparison with respect to nitrogen is somewhat obscured by varying numbers of applications).

No variation of response to nitrogen or to potash at different places on the same soil type could be proven, except that an exceptionally large response to nitrogen, 1.29 inches, was exhibited on the half of one new planting which had been burnt. (All of the inland new planting areas, except one half of this one experiment, were burnt; but none others showed this enhanced response).

Most of the response to phosphate is obtained with small applications: the average response to 3 oz. P₂O₅ per tree is 1.11 inches as compared to a maximum about 1.85 inches for 8 to 9 oz.

The phosphate response varies considerably for different inland soils. The estimated range of response is from .66 to 2.7 inches. No variation on alluvial soils is demonstrated.

Description of the Experiments

Since the response of a crop to fertilisers must be expected to vary with soil conditions on individual localities, recommendations for fertilising practice must be based on experiments at several places. If a series of such experiments are similar in design and accuracy, combination of results presents no great problem. But it is only recently that the value of having a large series of similar experiments has been fully recognised; and most of our knowledge has to be garnered from experiments of variable design and accuracy. The object of this paper is to consider what overall conclusions can be drawn from such a group of experiments on fertilising young rubber.

The experiments were laid out before a statistician was available in Malaya; at a time when agricultural experimenters had become aware of the value of randomized block and factorial designs, but before the principles underlying these designs had become understood by field workers. The main defect was failure to use blocks effectively for control of heterogeneity; further complications derive from incomplete factorisation of experimental variables, and irregular variation of the quantities of fertilizer applied. They are representative of much data which has to receive detailed attention from the statistician, since they have been costly to collect and it is not practicable to repeat the experiments.

Present data deal only with growth of the trees as represented by measurements of girth. Yield of rubber is of course the more important variate, but, owing to war, yield data are not yet available. For comparison of different clones (i.e. variety of planting material) yield records would be essential to draw conclusions of economic value. But as between agricultural treatments on the same type of planting material it may be assumed that there will be some degree of proportionality between size and yield. Furthermore, errors of yield observations being inflated by variability in tapping, girths may provide a more sensitive indication of response to fertilisers. They are also sooner and more easily measurable. The assumption that larger trees will continue to yield better must not be pushed too far since there are indications that phosphate may increase girth more than yield, that nitrogen may increase yield more than girth. However even in the absence of correlation between girth and yield in future years, rate of growth to maturity is still of sufficient economic value to merit investigation. Assuming

that rate of growth at maturity be 1.75 inches per annum (average of 14 experiments with range 1.3 to 2.2), that yield may be 1,000 lb. per acre per annum, and that the value of rubber less tapping and processing costs may be 20 cents per lb., then in the period before an unfertilised field would be ready for tapping, a fertilised field may already have earned \$115 per acre per inch of added girth.

Results from 23 experiments are presented. Eighteen are soils typical of the majority of inland rubber growing land of Malaya. Some of these are derived from granite, and some from shale or phyllites, usually containing appreciable amounts of ferruginous concretions ("laterite"). Four are coastal, of which three are on alluvial clay and one on thin peat over clay. One is on inland alluvium, not yet classified, but whose responses to fertilisers appear similar to those of the coastal alluvia.

Nineteen are areas known as "new plantings", by which is meant planting on virgin land newly opened from jungle. Four are "replantings", meaning plantings on land which previously carried a stand of rubber trees. It was expected in advance that the replanted areas would be more in need of fertilisers than the virgin land. On seventeen of the new plantings the jungle timber was burnt after felling, the exceptions being experiment V on coastal peat, and C (alluvial) which had much lalang: four (T, BA and M, inland, and O, coastal) had only light burns. The replanting experiments were unburnt.

Two experiments were planted in 1936, seventeen in 1937 and four in 1938. One (M) was planted with clonal seed, the rest were budded.* Replantings have only one clone each, but new plantings carry from two to five different clones. Each experiment with five randomized blocks has two clones confounded with blocks (two blocks carrying one clone, three blocks the other). The latin squares have either 3 clones confounded with rows of the squares, or have every column of the square split to carry five clones (or in the incomplete squares—experiment H—3 clones).

In conducting an experiment on the effect of fertilizers on a tree crop there are numerous complications not present with annual crops; in particular, the response may differ

* The normal procedure is to plant seed in the field. Between one and two years later, when 80 per cent. of the plants exceed four inches in girth, a bud from a "clone"—stems derived all from one original mother tree—is grafted into the cambium of each tree at about 4 inches from ground level. The original tree is then cut off just above the new bud which grows to form a tree of the given clone on a seedling rootstock.

according to time of application relative to the life of the tree, and the effects of applications at different times are not easy to sort out. These complications will not however be dealt with in this paper. In the series of experiments under consideration all experimental dressings began from the time of planting. An arbitrary manuring schedule was drawn up by the soils chemists, which consisted of beginning with light dressings applied twice yearly, later going on to annual dressings with increasingly greater amounts. Within any one series of experiments, every application maintains constant ratios of the ingredients. Having selected the basic units for each of the fertilisers to be tested a typical schedule was as in table 1.

Table 1. *A typical fertiliser schedule.*

Time of application No. of units applied	At planting 1	Years from planting			Years from cutting back				
		$\frac{1}{2}$ 1	1 (1)	$1\frac{1}{2}$ (1)	$\frac{1}{2}$ 1	1 2	$1\frac{1}{2}$ 3	2 4	3 (5)

In the original design, it being anticipated that budding would be done at, or soon after, one year of age, only two applications were scheduled for the seedling phase. In many experiments however growth was slower than anticipated, budding occurred anywhere between one and two years, and half yearly applications at the unit rate were continued up to 6 months before budding. Owing to the occurrence of war and occupation of Malaya by the Japanese, application of fertilizers was discontinued after November, 1941. This meant that most of the experiments (13) received applications of fertilizers only up to two years from cutting back. One was stopped after the application at one year, three after that at $1\frac{1}{2}$ years, three after 3 years. Three experiments did not pass through the pre-budding phase. Besides M with clonal seed, two (L and BA) were planted with budded stumps transplanted from nurseries. These received either 5 or 6 applications ending at $2\frac{1}{2}$, 3, or $3\frac{1}{2}$ years from planting.

Amounts of fertiliser applied, plot sizes, density of stand, mean girths at first post-war measurements and error variances are given in tables A and B. The records of amounts of fertilisers applied to replanting experiments, other than RW1, have been destroyed. It is assumed that RW3 and Sed would be like the 2³ new planting series, N⁴ like RW1.

Ages at times of observations here recorded varied from $6\frac{1}{4}$ to $8\frac{1}{4}$ years after cutting back rootstocks. Girth incre-

ments from 1946 to 1947, i.e. 5 to 6 years after the last application of fertilisers, suggest a tendency for growth on more backward treatments to be now slightly faster than on more advanced ones; but such differences are not always detectable, and over the period concerned never reduce established differences between treatments to any appreciable extent. We may therefore assume that estimates of responses are independent of age at time of observation.

The experimental designs fall into five series.

1. *Six-treatment series*: seven experiments with treatments (1), pk, n, np, nk, npk. Five of these are in latin squares, one in 4 randomized blocks, and one in two incomplete latin squares with the cleared jungle burned on one but unburned on the other.

2. *2³ series*: five experiments with the standard 2 x 2 x 2 factorial arrangement of presence and absence of n, p and k. Four of these have five randomized blocks. One has 8 blocks of 4 plots each with N.P.K confounded. (This originally belonged to the "Six-treatment" series, but was changed to this form after budding).

3. *3³ series*: two experiments of the standard 3 x 3 x 3 factorial arrangement in three blocks of 9 plots each. One of these with half acre plots has clonal seed. The other (on coastal clay) has 1½ acre plots each subdivided into three sub-plots of one half acre for different clones.

4. *Phosphate series*: five experiments each with five randomized blocks of the following eight treatments: (1) p₁, p₃, p₅, p₇, c, cp₃, np₅k, where the subscripts indicate the relative amounts of phosphate applied at each application, c = lime, and n and k as usual stand for nitrogen and potash.

5. *Replanting*: four experiments of factorial design.

N: 4 levels of n x 4 levels of p x 2 levels of k (no replication).

Sed. and RW3: 2 x 2 x 2 in n, p and k, 4 and 5 replications respectively.

RW1: 3 x 3 in n and p, omitting the zero level of both and with "basal" dressing of potash, 5 replications.

(RW1 and RW3 are adjacent and will be treated as one experiment, RW1 being used only to evaluate the response curve to varying levels of phosphate).

(The code of reference letters to individual experiments is given in the Annual Report of the Rubber Research Institute for 1939, p.83).

Each experiment has been analyzed individually, to provide estimates of the response to each ingredient and of

their interactions, along with estimates of their standard errors. As noted above, when these experiments were designed the principles underlying arrangement of plots to form blocks were not understood. The intention was to follow what appeared to be, and is in general, an approved principle, namely, that plots of a block should be contiguous. But, unfortunately, with the type of land available, and large plots of from one to two acres in size, this proved to be a snare, because it led to some plots being located in swampy areas, or intersected by a gully; to some plots of a block being on a hill top while others were on lower slopes, etc. The procedure to follow should have been to select for the formation of blocks plots conforming to some visible criterion of similarity—for example hill tops, aspect of slope, colour of soil, etc.—irrespective of relative position. To tamper with an experimental design once it has been laid out, or to reject some of the data, raises difficult questions of principle, and is likely to be condemned outright by some statisticians. The writer was originally inclined towards that attitude. But careful consideration of the data under review forces one to conclude that strict adherence to a principle of never rejecting data, or making adjustment for incidental circumstances, if it does not actually falsify the conclusions, would lead to such high errors as to mask conclusions to which the data quite patently point. If the experimental area contains one or two swampy or heavy clay areas on which the trees grow at a different rate from what they do on the bulk of the area, it may be a lottery as to which treatment or treatments fall on these areas, nevertheless a mean including one or two such plots when compared to a mean of plots all on well drained soil is not a reasonable estimate of the difference due to treatment on the same soil. If a planting area contains an appreciable proportion of heavy clay soil amid a dominating area of loam it may be reasonable to ask what is the average effect of fertiliser treatment over both soil types, but the answer to this question calls for replication of the treatments on both types with an estimate of error including interaction of treatments with soil types. To partially confound the effect of soil type with treatment differences can only produce biased estimates of the responses to both variables.

Thorough treatment of the problem calls for classification of each plot by soil characteristics, followed by fitting of constants for such characteristics as appear to have influenced growth. The writer has been able to visit only

two of these experiments since the war, and pre-war reports are mostly destroyed. Remedial measures have been sought only when it was obvious that variable soil conditions had had disturbing effects. In the majority of cases the cause of disturbance is known from memory of the areas or recorders' reports. A very few obvious discrepancies have been rejected on the evidence of the growth records alone. I am confident that subsequent inspection will reveal reasons for these.

When only one or two isolated plots are at fault the "missing plot technique" has been used. When a condition such as swamp or clay soil affects two or three contiguous plots, all have been adjusted equally by covariance on a dummy variate (for examples see Quenouille, 1948; Smith, 1949). When large areas are concerned resort has been had to direct fitting of constants by standard least squares procedure (e.g. BL, T and V). All three methods are theoretically equivalent; the variants of procedure being merely to facilitate computation.

In most experiments the number of trees per acre is fairly constant but in four experiments, for one reason or another, the numbers per plot have become sufficiently variable so that covariance of mean girth with number of trees per plot seemed worth examining. In three of these, adjustments for stand give a substantial gain in accuracy. The regression coefficients and reductions in error variance are shown in table 2.

Table 2. *Regressions on numbers of trees per acre.*

Experiment	A	V	RW1	C
Regr. coefficient ins/tree/acre ..	-.0229	-.0179	-.0288	-.0390
Standard error of regr. coef. ..	.00689	.00534	.00413	.025
Percentage reduction in error variance ..	26.5	24.7	57.0	Not Signif.

Theoretically regression on average space per tree per plot (= reciprocal of tree number) may be expected to be more nearly linear and may be preferred; but with the ranges of variation observed it is not possible to confirm this, and any difference in the results would be too small to make it worth while to convert observed numbers to reciprocals.

Not the least of the advantages of well-designed experiments with complete data is that whole series of treatment

contrasts are estimated with equal accuracy. This very greatly simplifies computations, of standard errors of group means as well as within individual experiments, and also the presentation of results. When data are incomplete, or have been adjusted in any way, we have the added complication that usually a separate error term has to be computed for each contrast within the experiment, and in consequence errors of group means have also to be computed individually. When the standard least squares procedure is used estimates of error for each effect at the place concerned are automatically indicated by the inverse matrix of sums of squares and products of the independent variates. When other computational procedures have been used the appropriate estimates of error have been derived as indicated by Smith (1949).

The *clone x fertilisers interaction* has been evaluated for 4 experiments, with results as in table 3.

Table 3. *Interactions: clones x fertilisers.*

Experiment	F 31	L	B	BA
Clone x Fert. mean square	[20]* 1.434	[20] .4110	[7] .4187	[7] .5954
Error mean square ..	[100] 1.653	[100] .3854	[21] .3652	[21] .5878

* Degrees of freedom in [].

F 31 and L have the most extreme range of clone types occurring in any of the experiments: the most vigorous and weakest differ by 9.8 inches at F 31, by 5.3 inches at L; or 36.4 and 24.4 per cent. of the general means respectively. In no case is interaction indicated. It is therefore assumed that for all experiments this interaction need not be separately evaluated and can be allowed to remain in the estimate of error.

ESTIMATES OF TREATMENT RESPONSES IN THE

SIX-TREATMENT SERIES.

One of the earliest experiments on fertilising rubber, to which much attention has been directed had six treatments similar to those of the experiments in this series. Its interpretation has given rise to much controversy. The problems which will be seen to arise in collating the results of such experiments present a strong argument for rigid adherence to strictly factorial arrangements for fertiliser experiments.

The treatments were:

		oz. per N	tree per P ₂ O ₅	unit. K ₂ O
(1): "control"	..	—	—	—
pk: concentrated superphosphate and sulphate of potash	..	—	.488	.600
n: sulphate of ammonia	..	.515	—	—
nk: sulphate of ammonia and sulphate of potash	..	.515	—	.600
np: "Nicifos B" = 67.5 per cent. sulphate of ammonia + 32.5 per cent. ammonium phosphate	..	.495	.495	—
npk: "Enpekay I" = 70 per cent. nicifos + 15 per cent. sulphate of potash + 15 per cent. muritate of potash	..	.500	.500	.600

Owing to shortage of Enpekay in 1941 only G and H received that mixture for their final dressings. On the others substitutes were applied as follows:

E (and F?): 33 per cent. Nicifos, 29 per cent. sulphate of ammonia, 15 per cent. concentrated superphosphate and 23 per cent. muriate of potash.

L and last two applications at D: 48 per cent. sulphate of ammonia, 25 per cent. conc. super. and 27 per cent. sulphate of potash.

F.31: 58 per cent. nitrate of soda, 11 per cent. Christmas Island rock phosphate, 11 per cent. concentrated superphosphate and 20 per cent. muriate of potash. Furthermore muriate of potash here replaced sulphate of potash in the nk and pk treatments.

If the different qualities of ingredients can be ignored, and responses interpreted as due only to the elements, N, P and K, although the experiments lack treatments p and k required for the standard 2 x 2 x 2 factorial design, the factorial approach can be used for interpretation, at least in part. The best method of combining treatments to estimate average responses is not however immediately obvious and depends on preliminary tests for existence of interactions.

The procedure here used is, in effect, to fit constants defined in the usual way for factorial experiments; that is we regard each treatment as made up of constants as in table 4.

Table 4.
(Constants)

Treatment	m	$\frac{1}{2}N$	$\frac{1}{2}P$	$\frac{1}{2}K$	$\frac{1}{2}NP$	$\frac{1}{2}NK$	$\frac{1}{2}PK$	$\frac{1}{2}NPK$
(1)	.. +	-	-	-	+	+	+	-
pk	.. +	-	+	+	-	-	+	-
n	.. +	+	-	-	-	-	+	+
nk	.. +	+	-	+	-	+	-	-
np	.. +	+	+	-	+	-	-	-
npk	.. +	+	+	+	+	+	+	+

Since there are 8 constants for only 6 observations some assumptions are necessary in order to obtain solutions. The 8 constants as assigned to these 6 observations fall into two independent groups; m, N, PK, NPK, and P, K, NP, NK. In the first group the obvious step to get a solution is to assume $NPK = 0$. This leads to N_1 and PK_n as defined in table 5. If in addition, we assume $PK = 0$, we get $\bar{N}$. In group 2, assuming $NK = 0$ leads to P_1 , K_n (= K in presence of n) and NR (defined in table 5) = NP; assuming $NP = 0$ naturally gives complementary definitions for P_n and K_1 , and NR remains the same but alters its interpretation from NP to NK. Assuming $NP = NK = 0$, gives $\bar{P}$ and $\bar{K}$. $\bar{N}$, $\bar{P}$ and $\bar{K}$ have variances equal to three quarters of the variance of a single treatment mean, the remainder have variances equal to that of a single mean. Of the five responses ($\bar{N}$, $\bar{P}$, $\bar{K}$, NR and PK_n) $\bar{P}$ and $\bar{K}$ are not independent, N and PK_n are not independent, all other pairs are formally orthogonal.

Table 5.

	(1)	pk	n	nk	np	npk
$4\bar{N}$	-2	-2	1	1	1	1
$4\bar{P}$	-1	1	-1	-2	2	1
$4\bar{K}$	-1	1	-1	2	-2	1
$2N_1$	-	-	+			+
$2PK_n$			+	-	-	
$2NR$	+	-	-			+
$2P_n$			-	-	+	+
$2K_n$			-	+	-	+
$2P_1$	-	+		-	+	

It will be shown that the factorial experiments indicate no interactions. Of the six-treatment series three show individually significant interactions, NR positive in all three, PK_n positive in one and negative in one. Over the whole group NR appears homogeneous and significant; PK_n appears zero except for one large positive value. If qualities of ingredients may have had some effect additional to effects of the main elements these may give rise to apparent interactions. If the quality effect throws out of alignment only one of the two treatments which are common to NR and PK_n then both will be affected, in the same direction by deviation of npk (*cf.* H, b), in opposite directions by n (*cf.* F). However the only effective difference of qualities appears to be the substitution of Nicifos B for sulphate of ammonia and superphosphate. This means that the interaction NR is completely confounded with the quality difference, Nicifos *v.* (sulphate of ammonia + super). If therefore, on the evidence of factorial experiments, we assume all interactions to be zero and introduce a constant for quality, this will be found to be estimated by $2NR$ of the factorial schema. The estimates of average effects will then be $\bar{N}$, P_1 , K_n and $Q = 2NR = npk - (n + pk - (1))$. This is not quite so pleasing as the former scheme, since P_1 and K_n appear with only three quarters of the accuracy of $\bar{P}$ and $\bar{K}$, and all three pairs between P_1 , K_n and NR are non-orthogonal. PK_n may still be evaluated also and interpreted as the difference in response to k in presence of nicifos and of sulphate of ammonia; as before it is not independent of $\bar{N}$ but is orthogonal with all others.

The effect of super alone can be estimated from

$P_s = pk - (1) - K = pk - (1) + \frac{1}{2}(n - nk + np - npk)$, but having a high error variance ($3V$, where V is the variance of a single treatment mean) is not likely to be of much interest. Sulphate of ammonia alone is obviously given simply by $n - (1)$, with error variance $2V$; or, introducing nk with a correction for K , by

$N_a = \frac{1}{2}(n + nk) - (1) - \frac{1}{2}K_n = \frac{3}{4}n + \frac{1}{4}(nk + np - npk) - (1)$ with variance $1.75 V$.

Mean effects $\bar{N}$, $\bar{P}$, $\bar{K}$ were selected as probably the best overall estimates, and averaged for the group, before it was considered that we had sufficient evidence to merit attention to a possible difference due to qualities. If the reader considers that the evidence warrants such consideration the alternative estimates can be derived as below, the formulae being applicable to group means as well as to individual experiments.

$$P_n = \text{nicifos} - S/A = \bar{P} + \frac{1}{2}NR : V(P_n) = V(NR) = 4/3 V(\bar{P})$$

$$P_l = \bar{P} - \frac{1}{2}NR : V(P_l) = \text{,,} \text{,,}$$

$$P_s = \text{super alone} = \bar{P} - \frac{1}{2}NR : V(P_s) = 3 V(NR) = 4 V(\bar{P})$$

$$K_n = \bar{K} + \frac{1}{2}NR : V(K_n) = V(NR) = 4/3 V(\bar{K})$$

$$N_a = S/A \text{ alone} = \bar{N} - NR : V(N_a) = 7/4 V(NR) = 7/3 V(\bar{N})$$

The formulae for standard errors will not be exact where there have been adjustments for tree density, swamps etc., but the alterations will be trivial and can be ignored. Whatever set of statistics is used makes little difference to the overall interpretation.

Two of these experiments have been subject for much cogitation: L because of controversy about response to fertilisers on coastal clay; H because it shows a unique response to nitrogen, to burning, and to Enpekey. Detailed results for these are given in table 6; the reader may judge for himself what conclusions are to be drawn.

Experiment L: For the general test of significance of treatment differences $F = 2.59$, ($n_1 = 5$, $n_2 = 20$), P ca. .06. As is well known, this overall test is not efficient to detect consistent effects of the same ingredient in different combinations. The two pairs of treatments giving an estimate of response to phosphate in the presence of nitrogen (nicifos v. sulphate of ammonia) indicate that this is highly significant, $P = .01$; but all fertiliser treatments have means lower than that of the control, and the effect depends on NR with an apparent depression due to sulphate of ammonia.

Experiment H: proves significant interaction of response to nitrogen with burning. The response to N on the burnt area is significantly greater than that on all other experiments except Sed. (replanting), despite the condition that all other inland new plantings were also burnt. The burnt area also shows the highest interactions NR and PK; both derive from the large girth on npk. This cannot be ascribed

Treatment	L	H (Unburnt)	H (Burnt)	H (B-U)
(1) = nil	22.25 ± .242†	21.141 ± .270*	19.523 ± .299*	
pk = Super + S/Pot	21.70	22.479 ± .269	20.973 ± .270	
n = S/A	21.44	20.941 ± .317	20.432 ± .299	
nk = S/A ± S/Pot	21.33	20.782 ± .269	20.048 ± .317	
np = Nicifos B	22.22	23.277	22.109 ± .269	
npk = Enpekav I	21.93	23.044	23.562 ± .273	
Mean	21.81	21.944	21.108	
Response NR	.514 ± .242s	.382 ± .279	.840 ± .272ss	.458 ± .389
PK _n	-.086	-.036 ± .284	.919 ± .301ss	.955 ± .413ss
$\bar{N}$	-.245 ± .209	.201 ± .235	1.290 ± .236sss	1.089 ± .333ss
N ₁	-.288 ± .242	.183 ± .279	1.749 ± .272sss	1.567 ± .389sss
N _a	-.759 ± .319s	-.181 ± .359	.450 ± .360	.631 ± .509
$\bar{P}$	.434 ± .209s	2.108 ± .240sss	2.176 ± .253sss	.068 ± .348
P _n	.690 ± .242ss	2.299 ± .284sss	2.595 ± .287sss	.296 ± .403
P ₁	.176 ± .242	1.917	1.756	.161
P _s	-.338 ± .418	1.535 ± .480ss	.916 ± .506	.619 ± .696
$\bar{K}$	-.462 ± .209s	-.387 ± .240	.114 ± .259	.501 ± .353
K _n	-.200 ± .242	-.196 ± .284	.534 ± .287	.730 ± .403
Enpk - N'fos = K ₂	-.292 ± .342	-.232 ± .382	1.453 ± .386sss	1.685 ± .543sss

† 5 per cent significant difference between any two = .713.

* Standard errors of treatment means are relevant only to comparisons within sections; since burnt and unburnt sections are not replicated no estimate of error is available for direct comparison of treatment means between sections. Consideration of fertility trends suggests that the difference unburnt *v.* burnt without nitrogen is real.

s indicates $P \leq .05$, ss: $P \leq .01$, sss: $P < .001$.

to a freakish result on one or two odd plots. All five plots of npk show less variation than the replications of any other single treatment, and rejection of lowest plots on other treatments could reduce average differences only by about .2 inch. To omit the two lowest plots of np would still leave npkb-npb = 1.10 inch. These interactions therefore appear due to "Enpekay" only, rather than to "Nicifos" which should also affect np. Whether it represents interactions of NPB, NKB, PKB, NPKB or of quality x burning cannot be determined.

Combination of Data

The procedure used to combine the data from different experiments follows the paper by Cochran (1937), which must be consulted for fuller explanation. The following notes are intended only as a brief explanation of the notation to be used.

"The most general hypothesis to be considered is that the treatment response at any centre is the sum of two parts, each normally and independently distributed; one, representing the contribution of local experimental errors, varies about zero with standard deviation σ_i , while the other, which represents the responsiveness of the centre to the treatment, varies about a general mean μ with standard deviation $\sigma(\mu)$. The parameters μ and $\sigma(\mu)$ are the same for all centres, but σ_i varies from centre to centre. An estimate s_i of σ_i , based on n_i degrees of freedom, is available from the local analysis of variance." (Cochran, 1937).

Procedure is greatly simplified if it can be assumed either that the estimates of response have equal accuracy in every experiment, or (if numbers of replications vary) that error variances per plot are equal. In the first case obviously problems of weighting do not arise, in the second weighting will be proportional to numbers of replications and an ordinary pooled analysis of variance will be in order. But since the experiments here under review have varying designs, sizes of plots and numbers of replications, it would be unreasonable to suppose that the true error variances were equal, except fortuitously. They will therefore be assumed, without test, to be heterogeneous.

Responses to each fertiliser ingredient are considered separately. The first step is to determine whether or not it may be reasonable to suppose that they have been similar at all places. To test this, with heterogeneous variances,

the most efficient statistic at present available is that designated Q by Cochran (*loc. cit.*): $\chi^2_{w'}$ is a transformation of Q which may be referred to the ordinary χ^2 tables—it is always less than Q if Q is greater than 5/7ths of $(k-1)$ therefore need only be computed if Q itself suggests significance.

An alternative statistic, denoted F' , is simply the ratio of the unweighted mean square between responses at different places to their mean error variance. It is simpler than Q to compute, but is less efficient when variances are markedly heterogeneous. Usually if a clear cut answer is indicated by F' there is no need to evaluate Q , however in the following, except for responses to phosphate, Q is given throughout for the sake of uniformity in presentation. Responses to phosphate obviously vary, and so F' is there more useful since it leads directly to an estimate of the variance between places.

When error variances are heterogeneous and it is permissible to assume no interaction of responses with places, the most efficient estimate of mean response is given by the weighted mean, denoted $\bar{x}_w$, using as weights the reciprocals of s^2_i (hereafter denoted by V_i).^{*} Its standard error is computed by a formula derived by Miss Sarah Porter and G. S. Watson, using in part the results of a sampling experiment to ascertain the accuracy of $\bar{x}_w$ relative to a weighted mean using the true but unknown weights $(1/\sigma_i^2)$: as yet unpublished.

When interaction with places has been demonstrated the average response may be estimated from the unweighted mean ($\bar{x}$) or (provided responses and precisions are uncorrelated—cf. Yates and Cochran, 1938) from the “semi-weighted” mean ($\bar{x}_{sw}$). (See discussion of phosphate effects).

t' = the ratio of a mean response to its estimated standard error, is used to test whether or not the mean response differs significantly from zero.

* If all experiments of a group are of similar design and data are complete, the weights for all responses in each experiment, and thence error variances of group means, will be proportionate. The saving of computing labour thus made possible is well worth bearing in mind during planning and execution of a group of experiments.

F' and t' will be distributed as Fisher's F and t only if all σ_i are equal. For variable σ_i , their exact distribution has not yet been obtained. As an approximation we calculate n_e as a quasi-number of degrees of freedom such that the denominator of F' or t' , divided by its expected value, may have a distribution with mean and variance approximately equal to that of a χ^2 distribution with n_e degrees of freedom. It is used as argument with which to enter the standard F and t tables (Smith, 1936; Cochran, 1937; Satterthwaite, 1946). The formula for n_e to be associated with t' has been evaluated only for the unweighted mean. The appropriate formulae for $\bar{x}_{lv}$ and $\bar{x}_{slv}$ are not yet known.

Other notation used is as follows:

k = the number of experiments in any group under consideration.

$\bar{n}$ = the mean of the numbers of degrees of freedom for the estimate of error in each of the k experiments. Usually in this paper the harmonic mean is quoted.

$V(\bar{x})$ = the error variance of the mean response, etc.

$\bar{V}$ = the average error variance of the responses in each of the k experiments = $\Sigma V_i/k$.

Confidence intervals quoted are intended to be ninety five per cent confidence intervals, but they can here be only approximations since the correct distribution of t' is not known. The interpretation of a strict confidence interval is that, when it is stated that the true value of the response lies somewhere in that interval, 95 per cent of such statements will be true, 5 per cent will be false.

Most experiments were planted with approximately 180 trees per acre. Four had appreciably different planting densities (viz. G, 270; M, 217; H, 140; Sed. 132); but it is not possible to assess the effect of these variations on response to fertilisers. Fertilisers were applied in given amounts *per tree*, and it is assumed for the purpose of this report that responses on this basis may have been independent of space per tree within the range of variation covered by these experiments.

INTERACTIONS (*Data in tables C and D*).

It is obvious from table C that interactions on alluvial soils are not detectably different from those on inland soils, but those determined from the six-treatment series seem to

Table 7. *Interactions.*

Factorial experiments				Six-treatment series		Six-tr. series omitting H(b)	
NP	NK	PK	PK _n	NR	PK _n	NR	NR
No. of expts. (<i>k</i>)	9	9	8	8	7	7	7
$\bar{n}$ (Harm. mean)	22.6	22.6	21.1	21.1	—	—	—
<i>Q</i>	6.678	9.171	4.614	9.336	6.625	6.421	6.421
$\chi^2_w (k-1 \text{ d.f.})$	—	8.333	—	8.337	—	—	—
Weighted mean ($\bar{x}_w$)	.052	.005	.016*	.405ssst	— .195	.345ss	.345ss
$\pm$	$\pm .085$	$\pm .085$	$\pm .105$	$\pm .104$	$\pm .111$	$\pm .111$	$\pm .111$
$t' (\bar{x}_w)$	.612	.060	.187	3.888	—1.758	3.102	3.102

† Excluding L (on alluvial soil), $x_w(\text{NR}) = .3849 \pm .1131$ is the figure appropriate to adjust main effects on six-treatment experiments on inland soils if it be desired to make allowance for possible quality effect of nicifos.

* Average PK for all 17 experiments is $\bar{x}_w = -.019 \pm .0662$.

merit separate consideration. Treating the two halves of H as separate experiments, summary statistics are presented in table 7.

All interactions of the factorial experiments appear homogeneous with mean approximately zero.

In the six-treatment series heterogeneity of PK_n (P ca. .016) appears due entirely to the contrast between H burnt area, $.919 \pm .287$, and the remainder which have mean $-.195 \pm .111$ (P for difference from zero ca. .08). Pending further evidence it seems best to assume $PK = 0$.

NR appears homogeneous (P ca. .3), and the mean is very highly significant ($P < .001$). For reasons given under discussion on the interpretation of these six-treatment experiments, the simplest explanation seems to be that nicifos gives better results than super and sulphate of ammonia, the estimate of this difference being $2NR = .81 \pm .21$ inch. Present evidence can however only be taken as suggestive for further investigation with experiments expressly designed to separate effects of qualities and of interaction.

Further data on interactions will be noted under discussion of the effect of potash.

It is obvious from table D that there is no demonstrable interaction of phosphate x lime. Further computation would be waste of time.

MISCELLANEOUS EFFECTS IN THE PHOSPHATE SERIES (*Data in table D*).

It is obvious that there is no consistent response to *time*; and, since the response on the peat-clay is the median of the five, no test is required to show that we cannot distinguish responses on the two soil types. For all five $Q = 7.158$, $\chi^2_w = 6.446$, P ca. .16; and the weighted mean is $-.005 \pm .13$ (approximately).

Potash and *sulphate of ammonia* jointly appear to have been harmful. Responses on the four inland soils are homogeneous, $Q = 1.362$, $P > .7$. The response on the peat clay appears to be approximately zero, but it is not significantly different from the weighted mean of the inland soils, the difference being $.375 \pm .470$. The overall weighted mean is $-.372 \pm .172$, $P < .05$.

In view of other results for N and K, the negative response is presumably due to K. Therefore the means of each group separately will be present in table 9 for com-

parison with direct estimates of effect of K in other experiments.

RESPONSE TO POTASH (*Data in table E*).

Total amounts of potash applied in the different groups are very unequal. The units vary from .336 oz. K_2O per tree in the 2³ series to .600 oz. in the Six-Tr. series; besides variation in number of units applied as indicated in Table A. The total amounts per tree are indicated in Table E, showing average amounts applied to each group as

Six-treat. group	..	9.1	oz. per tree
2 ³ and J	..	3.7	„ „
Replanting	..	2.7 ?	„ „
Alluvial	..	4.4	„ „
{Phosphate Series	..	1.56	„ „

It does not appear possible to detect any correlation between response and amounts used, and there is no way to determine what adjustments should be applied. There seems therefore nothing to be done except to treat quantity applied as an uncontrolled variable. Since the response, if any, is harmful, this raises no difficulty in assessing the economic value.

Table 8. *Potash.*

Group	Inland soils			Alluvial and clay soils
	Six-tr.	2 ³ & M	Repl.	
No. of expts. (<i>k</i>)	6	4	3	4
$\bar{n}$ (Harm. mean)	.. 20.5	23.7	21.1	20.3
<i>Q</i>	.. 5.441	1.739	.980	2.049
χ^2_w	.. 4.925	—	.973	—
W'ted mean (x_w)	— .058*	.077	— .246	— .442sss
$V(x_w)$	.. .00973	.01397	.03948	.01096

Pooled means: Inland new planting ($k = 10$) — .004 $\pm$.077
 All inland ($k = 13$) — .034 $\pm$.072

* Estimate of response in the six-treatment series, freed of some possible response to qualities of associated *n* and *p*, is $K + \frac{1}{2}NR = .134 \pm .113$.

The data are summarised in table 8. Responses are homogeneous within groups, and no differences of mean responses among the three inland groups can be detected. The difference mean inland — alluvia is .408 $\pm$.127, $t' = 3.208$, $P < .01$ (for any $n_e > 18$).

Table 9. *Comparison of potash effects.*

	Phosphate Series (M)	Other Experiments (K)	Difference	Mean
Inland	.. - .424	- .034	+ .390 $\pm$.198	- .083 $\pm$.0672
Peat-clay or Alluvial	.. - .049	- .442	- .393 $\pm$.444	- .423 $\pm$.1018
Variances	.. .03416	.00521	.0394	.00452
	.1864	.01096	.1974	.01035

Assuming that the negative response (M) to n and k together in the phosphate series was due to potash, that series appeared to tell a different story, indicating that potash was harmful on inland soils, while the effect was not shown for a coastal peat-clay soil. Are these results from the two series discordant? The relevant weighted means are collected in table 9.

As a measure of disagreement between the two sets consider the difference of differences (interaction of a 2×2 table). This is $.783 \pm .4866$, and the two sets of data are not proven to disagree; but the response M on peat-clay, being estimated from only one experiment, is poorly determined and contributes little information. The difference for inland soils in the two groups, $.390 \pm .198$, is almost significant. The combined means for all experiments still indicate significant negative response on alluvial or clay soils, not proven for inland soils. The average difference between major soil types, $0.340 \pm .122$, $t = 2.787$, is still significant.

Potash in presence of n and p: It has been observed in two or three experiments with miscellaneous fertiliser combinations that the complete fertiliser, npk, has given the highest yield of rubber, and it is a commonly held opinion that potash may have a beneficial effect when in combination with phosphate and nitrogen, although not giving any response, or possibly a harmful one when alone. The argument is usually applied to yield, rather than to growth, but the same type of observation appears markedly in the girths for H (burnt), table 6. It has therefore seemed worth while, since for the six-treatment series the three factor interaction cannot be evaluated and the two factor interactions are confused, to add to table E the difference between the two treatments npk and np in each experiment. These differences estimate response to potash in presence of nitrogen and phosphate; or, more specifically, in the six-treatment series in the presence of nicifos, in the others in presence of super, Christmas Island rock phosphate and sulphate of ammonia. They are analysed in table 10. Except for the isolated result shown by H burnt area, the observations are clearly homogeneous within groups, and the means are not different from the estimates of average response to potash (table 8).

Response to potash in presence of p and n together, but not with one or neither, implies a three factor interaction which is rarely detectable in agricultural experiments. For the eight factorial experiments for which NPK can be

Table 10.
Response to potash in presence of nitrogen and phosphate (K₂).

In presence of		Nicifos		Rock, Super & S/A		Alluvial soil	
No. of experiments (<i>k</i>)		7†	6‡	7		4	
$\bar{n}$ (harm. mean)		21.4	20.5	23		8.4	
<i>Q</i>		19.156*	4.115	2.433		.078	
$\bar{x}_w$		..	.067	-.162 ± .171	.175 ± .175	-.361 ± .197	

* $\chi^2_w = 16.497$, *P* ca. .012.

† including H burnt and unburnt areas as two experiments.

‡ omitting H burnt.

estimated, the weighted mean is $.100 \pm .104$; of the individual values three are negative and five positive, six were less than their standard errors and two slightly greater. It has already been shown (table 7) that no two factor interactions with potash can be detected. Therefore except for the one isolated observation of growth with Enpekey on H burnt area, there is no evidence that potash affects growth differently whether other fertilisers are present or absent. On alluvial soils it appears harmful either way.

RESPONSE TO NITROGEN (*Data in table F*).

The unit dressing is approximately constant at 0.5 oz. N per tree, except in experiment M whose unit was .206 oz. (table B). But the numbers of applications vary (table A), and a graph of responses against numbers of units suggests that magnitudes of response are affected by total quantities applied. However, complications associated with the different soil types, and erratic results shown by experiments with high errors—Sed, G and M, make it impracticable to assess from these data the real effect of altering numbers of units with different frequencies of application. About all that can be deduced from the graph is that the observations do not disagree with a hypothesis that response may be proportional to total amounts applied. Accordingly the following analyses are based on estimates of response to 12 units (N_{12}) derived by multiplying each observed response by $12/u$, where u is the number of units applied to each experiment (table A). (Variances being multiplied by $(12/u)^2$, an effect of this adjustment is to give lower weight to experiments with fewer applications).

Since new planting inland experiments averaged 12 units, while the replanting experiments had perhaps 5, 8 and 8 units respectively (uncertain owing to wartime destruction of records) the contrast between these two groups cannot be satisfactorily assessed.

Another complication derives from the results of experiment H which have been discussed in detail above. Interaction of N x burning seems proven beyond reasonable doubt, and response to N on the burnt area seems undoubtedly larger than any other observed on new plantings. But, in that the estimated mean response is entangled with interactions and/or quality effects, it is not clear what figure would be "fair" for comparison with other experiments. The response to N on the unburnt area is similar to responses on other experiments, but all other new planting experiments

Table 11. *Nitrogen.*

Group	Inland soils				All Inland	Alluvial & clay soils
	Six-tr.	23 + M	Replanting			
No. of experiments (<i>k</i>)	5	4	3	12	4	
<i>Q</i>	2.427	2.278	1.033	7.216	5.692	
Weighted mean ($\bar{x}_w$)	.305*	.130	.398	.259ss	.021	
	± .093	± .139	± .338	± .0772	± .135	
$V(\bar{x}_w)$	.008691	.01936	.1143	.005958	.01832	

* The six-treatment series had a larger number of applications than the others, and combined with doubt as to the possibility of a quality effect it is difficult to decide on the best estimate for comparison to other experiments. Some alternative estimates are as follows:

$\bar{N}$.. as observed $.360 \pm .115$, adjusted proportionately to 12 units $.305 \pm .093$.

N^a (S/Am. only) as observed $.065 \pm .175$, adjusted proportionately to 12 units $.083 \pm .142$.

The difference between $\bar{N}$ and N^a is due rather largely to one experiment (F) with high value for NR and lowest error.

(NR = $\bar{N} - N^a$ - Na differs from the value given in footnote to table 7 owing to weighting for the total amounts of fertilisers applied, and to omission of experiment H).

on inland soils were burnt. Therefore it seems best to regard experiment H as a "group" by itself. Its data are presented in table 6 and omitted from the summary of the remainder, table 11.

In absence of H(b), the next highest response, $.71 \pm .65$ at Sed. (replanting) for 8(?) units could be ascribed to high experimental error, and we would conclude that no differences in response of inland soils can be detected. In view of H, the Sed. result may be real, and judgement must be reserved. It appears possible that a few odd soils may respond more to nitrogen than the majority, but present evidence is not conclusive. H indicates that a nitrogen deficiency may be markedly enhanced by burning.

The responses are summarised in table 11. No differences between experiments can be detected. The average response on inland soils appears highly significant ($t = 3.355$, P nearly .001), no response on alluvial soils is indicated. The difference inland—alluvial, $.238 \pm .156$, is not statistically significant, but seems indicative of a real difference.

RESPONSE TO PHOSPHATE (*Data in table G*).

The total amounts of phosphate applied to each experiment vary considerably, but show no correlation with magnitudes of responses. Apparently the increase in response for successively higher dressings in the range concerned is small relative to variation between soils. Thus in the phosphate series increase from $p = 5$ to 7, corresponding to 5.85 to 8.19 oz. per tree, increased girth, on the average, only by about 0.1 inch, whereas for experiments on inland soils the total responses vary over 1.4 inches. Therefore we are forced to regard size of dressing as a random variable having little effect.

For experiments with several levels of phosphate the estimated responses to the maximal dressings are quoted, as these correspond most closely to the dressings on other experiments with only one level.

Variation of responses to phosphate from place to place is sufficiently proved by the variance ratio F' . Variance of the responses, $s^2(\mu)$, is estimated by the difference between the unweighted variance of observed responses and their average error variance ($\bar{V}$). Table 12 gives these statistics for each group of experiments.

It is obvious, without critical test, that responses are different on alluvial and on inland soils. The two types will therefore be considered separately.

Table 12. *Phosphate.*

	Inland soils					Alluvial & peat-clay
	Six-tr.	2 ³ + M	Phos.	Repl.	Total	
<i>k</i>	6	4	4	3	17	5
$\bar{n}$ (harm. mean)	20.5	23.7	24.5	21.1	—	21.7
Total <i>n</i>	125	99	101	66	391	114
<i>n_e</i>	75	59	94	39	205	87
Mean square between expts.	.2058	.3922	.5807	.2709	.3695	.03751
$\bar{V}$	.0708	.0594	.1356	.2015	.1064	.08003
$s^2(\mu)$	.1350	.3328	.4451	.0794	.2631	—
$s^2(\mu)/\bar{V}$ ($=F' - 1$)	1.9 _s	5.6 _{ss} +	3.3 _{ss}	.39	2.47 _{sss}	—
Arith. mean ($\bar{x}$)	1.519	1.498	2.216	1.746	1.718	.205
s.e.	± .185	± .305	± .381	± .300	± .147	± .127

s indicates $P < .05$, ss: $P < .01$, ss+: $P < .005$, sss: $P < .001$.

Standard errors of arithmetic means are computed from the mean squares between experiments, and are therefore based on few degrees of freedom; except for the last group where $s^2(\mu) = 0$.

Alluvial Soils. There is no evidence of interaction with places, and, as error variances differ considerably (from .025 to .19), the weighted mean is the best overall average. This is $\bar{x}_w = .1775 \pm .1023$; it is not significantly different from zero, but does fall in the 5 per cent. tail in the expected direction.

Inland soils: Variation of response between places is significant for all groups of experiments except the replanting ones and lack of significance here does not indicate any different behaviour because, with only 3 experiments, the variation is masked by an exceptionally high error in one. (The difference between the two more accurate ones is $.896 \pm .423$). Unweighted analysis of variance within and between groups gives.

	d.f.	M.Sq.	
Between groups	.. 3	.4743	$P > .25$
Within groups	.. 13	.3453	

Since the extreme group, the phosphate series of experiments, has a relatively high error, consideration of weights could show only less difference between group means. The replanting group, which might be considered separately on a priori grounds, shows an average response intermediate among the others. There are therefore no grounds for suspecting any differences of behaviour as between types of experiments and the whole 17 may be considered together.

Since $s^2(\mu)$ is clearly not zero, the weighted mean is not a suitable estimate of the average response. The most efficient available estimate of the average appears to be the semi-weighted mean

$$\bar{x}_{sw} = \frac{\sum \left(\frac{x_i}{s^2(\mu) + V_i} \right)}{\sum \left(\frac{1}{s^2(\mu) + V_i} \right)}$$

this being a sample imitation of the efficient estimate which would be given by replacing $s^2(\mu)$ and V_i by the population parameters $\sigma^2(\mu)$ and σ^2_i , which of course are unknown. However it has been shown by workers at North Carolina that when $n = 20$ the unweighted mean ($\bar{x}$) has efficiencies relative to $\bar{x}_{sw}$ of

96.9 per cent when $\sigma^2(\mu)/\sigma^2_i = 2$ and σ^2_i vary in ratio $\frac{1}{2}, 1, 2$.
 95.7 " " " " " " " " " " " " $\frac{1}{4}, 1, 4$.

Our estimate of $\sigma^2(\mu)/\sigma^2_i$ is $s^2(\mu)/\bar{V} = 2.47$, and the variation of V_i is intermediate between these two cases, so either statistic may be considered suitable. They are very similar:

$$\bar{x} = 1.718 \pm .147$$

$$\bar{x}_{sw} = 1.686 \pm .144^* \quad (\text{See footnote on page 156}).$$

If we may assume that responses over inland soils at large may be approximately normally distributed, the indications are that about 95 per cent. of places might give responses in the range $1.686 \pm 2\sqrt{.263}$, i.e. between .66 and 2.71 inches; but these limits must be regarded as rather poorly determined. (The range of observed values on 17 experiments was from .70 (experiment A) to 2.35 (N) or 3.31 (T), cf. notes on T below).

REGRESSIONS ON LEVELS OF PHOSPHATE (*Data in table H*).

For all the experiments with varying levels of phosphate the observations can be satisfactorily fitted by a quadratic regression; variance of deviations therefrom being in no case greater than experimental error variance determined from replicates. To make results of different experiments comparable the levels of phosphate applied are expressed in units similar to those used in the phosphate series, that is the basic level ($p = 1$) is taken as .09 oz. P_2O_5 per tree per unit application. The number of units applied over the period of manuring varied from 9 to 13, but the experiments mainly concerned in the following discussion had $12\frac{1}{2}$ to 13 units. The effect of varying amounts of successive applications requires further investigation.

Table H gives the regression coefficients. P' is the linear regression coefficient, that is the average increase in girth throughout the observed range per "dose" of p (i.e. .09 oz. P_2O_5 x the number of temporal units applied in each experiment). Although normally the most accurate measure of average increase per dose this coefficient is not altogether satisfactory for comparisons between these experiments because the distribution of observed levels of p is not the same in all, in consequence of which P' represents slope of the quadratic regression at varying levels, p' , as indicated in the last column of the table. The coefficients $b_{3.5}$ and c are as determined for quadratic regressions in the form $g_p = g_{3.5} + b_{3.5} (p - 3.5) + c (p - 3.5)^2$ i.e. $b_{3.5}$ is the slope at $p = 3.5$, or approximately the increase in girth in changing the level of p from 3 to 4; and c is a measure of the curvature or of the change in response for each succeeding half dose of p . The level $p = 3.5$ is chosen

* The standard error of $\bar{x}_{sw}$ is estimated from

$$V(x_{sw}) = \left\{ \sum \frac{1}{s^2(\mu) + V_i} \right\}^{-1}$$

This formula seems likely to tend to underestimate it, but no more efficient estimate has yet been devised.

as being near the point where slope is most accurately evaluated on the average of all experiments, and therefore most suitable for comparisons.‡ Table 13 gives analyses of variance of the coefficients for the five new planting experiments on inland soils. On four experiments the regressions are very similar.†

The fifth experiment, T, did not receive the last dressing of 4 units and it is difficult to decide on what basis it should be compared with the others. For the tests of table 13 no adjustment has been made so that the coefficients represent response to smaller doses of p. Since even so b is larger than in other experiments, and adjustment could only make it still larger, it is demonstrated that response to phosphate is here greater than usual. No difference of average curvature from that of other experiments can be demonstrated either with or without adjustment. Within experiment T there are considerable differences of fertility associated to some extent with proximity to swamp; and graphs of the

Table 13. *Regressions on levels of phosphate.*

	Excl. T. $b_{3.5}$	Including Expt. T* $b_{3.5}$ c
No. of expts. (k) .. 4	5	5
Mean square between expts. ..	.001070	.009492 .0000876
V ..	.002266	.002631 .0005667
P' ..	.472	3.608 1/6.46
P .. > .5	< .02	> .95

* not adjusted for number of units applied.

‡ The slope at any other point, p , can be estimated from

$$b_p = b_{3.5} + 2c (p - 3.5) = P' + 2c (p - p')$$

and its error variance from

$$V(b_p) = V(P') \times 4 (p - p')^2 V(c),$$

P' and c being orthogonal. (In experiments P and U, where values for some abnormal plots were replaced by "missing plot" estimates, P' was computed as for complete data; therefore in these P' and c are not strictly orthogonal—and hence $V(P') > V(b)$ in experiment U—but any error in applying the above formula for $V(b_p)$ will be trivial).

† Curvatures, indeed, appear more similar than they ought to do relative to their estimated errors—omitting T, see below, P is greater than .99. In seeking some possible explanation it has been noticed that three experiments have the same plot arrangement and this forms an incomplete latin square with three columns missing, whereas when computing errors it was assumed that the columns formed simple randomised blocks. Further rather complex computations to ascertain whether or not this may explain the apparently freakishly close agreement have not yet been made.

regressions for restricted groups of plots suggest that the response to phosphate may vary like a set of Mitscherlich curves starting from different levels.

Responses on replanting experiments are obviously similar. Since, as already shown in table 12, total response on alluvial soils cannot be detected as significant, curvature is naturally also undetectable. Computations for these experiments are not yet completed. It is intended to do considerably more work on problems arising in comparison of these curves. The present report is tentative.

CONCLUSIONS

The data reported here are only for effect of fertilisers, applied at planting and during the first $2\frac{1}{2}$ to four years of life according to schedules and in amounts as indicated in Tables 1, A and B, on the girth attained at 6 to 8 years after cutting back the seed stocks. Twenty three experiments are summarised, eighteen having been on inland soils, five on alluvial or peat-clay soils.

No response to *lime* can be detected, either with or without phosphate. (A slightly harmful effect during the first year of growth was indicated, but was not sustained).

No interactions of fertiliser ingredients can be detected. Confidence intervals are

PK - .15 to .12 inch

NK - .16 to .18 inch

NP - .12 to .22 inch

In the "Six-treatment" series of experiments some interaction of N with P and K together ($NR = .4 \pm .1$ inch) was indicated. Since in these experiments the different fertiliser combinations varied also in the kinds of chemicals applied, a possible explanation is that this result may be due to a contrast of qualities of fertiliser, but with the data available this cannot be separated from interactions.

The amounts of *Potash* applied varied greatly from one experiment to another, 2.2 to 11 oz. K_2O per tree, but this has little effect on the conclusions. No response can be demonstrated on inland soils, $K = -.034 \pm .072$; on alluvial soil it appears to be harmful to growth, $-.442 \pm .105$ inch. Confidence intervals:

inland .. - .18 to .11 inch.

on alluvium .. - .65 to - .23 inch.

Four experiments on varying levels of phosphate on inland soils had an odd treatment, np_5k , giving an estimate of response

to nitrogen and potash together (1.34 oz. N + 1.56 oz. K_2O per tree) in the presence of phosphate (5.85 oz. P_2O_5 per tree). This produced a response similar to that for potash on alluvial soils in the other experiments, viz. - $.424 \pm .185$ inch.

There is no evidence that potash is more beneficial when phosphate and nitrogen are both present, than when one or both is absent.

Nitrogen: One experiment (H) on which felled jungle timber was burnt on one half of the area and not burnt on the other half showed a marked interaction of nitrogen x burning. Otherwise differences in response at different places on similar soil types could not be detected, nor could a difference be proven between new planting and replanting. The difference between inland and alluvial soils is not statistically significant but is probably real. Average responses to about 6 oz. N per tree were.

	Average	Confidence interval
Expt. H'		
(burnt area) ..	$1.290 \pm .236$ inch	.82 to 1.76
Expt. H		
(unburnt area) ..	$.201 \pm .235$	- .27 to .70
Other experiments		
on inland soils*	$.259 \pm .077$	.10 to .41 inch
Alluvial soils ..	$.021 \pm .135$	- .25 to .29

* New plantings all burnt, replantings unburnt.

The amount of *Phosphate* applied on different experiments varied from 5.3 to 9.6 oz. per tree, but there seems little difference in the response to these levels.

For five experiments on alluvial soil no variation of response between places could be demonstrated. The average response was $.178 \pm .102$ inch, $P < .1$. Although not proven significant this lies in the 5 per cent. tail in the expected direction.

On inland soils there was considerable variation in response from place to place, the standard deviation between places being estimated at .513 inch. The average response was $1.686 \pm .144$ inches, with confidence interval 1.40 to 1.97 inches. For individual places about 95 per cent. may be expected to show responses between .66 and 2.71 inches. No difference between replantings and new plantings was indicated.

Response to varying levels of phosphate could be evaluated for 9 experiments, but only the 7 on inland soils are of interest. The curves for four of these (new plantings) can be described by the equation

$$g = g_0 + .4515 p - .02761 p^2$$

for girth (g) in inches and for p in units of one oz. P_2O_5 per tree. One new planting showed less falling off at higher levels. One replanting showed a higher initial response with quicker falling off, one showed lower initial response but slower falling off.

The average regression indicates increases in girth for varying amounts of phosphate as follows.

P_2O_5 oz./tree	..	1	2	3	4	6	8	10
Girth increment	..	.424	.793	1.106	1.364	1.715	1.845	1.754

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Soil Type	Series	Expt.*	Total no. of units applied	Plot size acre	No. of trees at planting	per acre 1946	n	V (ins ²)	Mean (ins)	C.V. %
Inland	..	Six-tr.	17	1.5	180	160	20	.4643	25.15	2.71
		E	14	1.5	180	149	20	.3833	21.30	2.91
		D	12	1.5	140	139	27	.3497	21.52	2.75
		H	18.4	2.5	270	137 ^t	18	.9198	28.35	3.38
		G	12	1.5	180	177	20	.2261	23.35	2.04
		F								
		F31	18	1.08	180	125 ^t	20	.7032	26.99	3.11
	2 ³	A	10.5	1	176	146	27	.3967	23.22	2.70
		B	11.5	1	176	148	28	.3786	23.57	2.62
		BA ^b	7.5	1	?	166	27	.4371	22.53	2.93
	3 ³	M ^c	12.5	0.5	217	208	17	.4794	23.68	2.95
	Phosphate	T	9	1	176	156	30	.7192	21.56	3.94
		U	13	1.25	180	162	27	.3390	21.46	3.07
		P	13	1.25	185	170	25	.2469	22.68	2.19
		BL	13	1.25	180	176	19	.2820	24.24	2.18
	Replanting 4 x 4 x 2									
	2 ³	N	5?	0.75	180	165	17	.6440	19.86	4.04
	2 ³	Sed.	8?	0.5	132	114	21	.34058	18.88	9.77
	2 ³	RW3	9.3?	0.5	?	154	28	.7150	23.07	3.66
	3 ²	RW1	8	0.5	?	154	35	.3163	23.63	2.38
	{Six-tr.	Lb	12.5	2	180	179	20	.3506	21.81	2.72
Coastal	..	K	6.5a	1.5	180	163	18	.2027	24.93	1.81
Alluvial	..	O	9.5?	1.5	180	159	18	.1285	21.73	1.65
Clay	..	V	9	1.25	185	153	30	.6282	24.61	3.22
Peat over clay	..	C	9.5	1	180	165	28	1.1341	22.16	4.81
Inland Alluvium	2 ³									

b = Planted with budded stumps.

c = Planted with clonal seed. Other experiments budded in the field at one to two years.

? = Uncertain owing to destruction of records during Japanese occupation.

a = after budding only. Pre-budding 3 units on six-treatmentscheme. After budding new blocks were formed and new treatments randomized with respect to previous ones.

RW1 = in the same field as RW3, with levels 1, 2 and 3 of p and n and no unfertilized plots, is used here only to estimate regression on phosphate levels.

n = number of degrees of freedom for estimating error variance.

V = error variance per single plot.

C.V. = coefficient of variability per single plot = $\sqrt{V}/\text{Mean}$.

t = recently thinned out.

* = A key to the location of each experiment is given in the Annual Report of the R.R.I.M. for 1939, facing p.82. Three not there recorded are Sed., central Johore; BA, north Johore; F31, R.R.I. Expt. Station, hill area.

Table B. *Units of fertilisers in oz. per tree.*

Series	N	P ₂ O ₅	K ₂ O
Six-treatment ..	.506	.494	.600
2 ³ ..	.494	.812	.336
3 ³ M ..	.103 x 2	.383 x 2	.160 x 2
3 ³ O ..	.258 x 2	.289 x 2	.240 x 2
Phosph. ..	.103	.09 x 7	.12
3 ² RW1 ..	.27 x 3	.27 x 3	.26 (on all plots)

Table C. *Interactions of N, P and K.*

Expt.	NP or NR§	PK†	NK	Variance
<i>Inland Soils</i>				
Six-tr. E	-.050	.173		.07738
D	-.012	-.458		.06388
H(u)	.382	-.036		■
H(b)	.840 _{ss}	.919 _{ss}		■
G	.205	-.101		.22995
F	.551 _s	-.488 _s		.03767
F31	.630	.140		.11720
2 ³ A	.369	.241	.278	*
B	-.243	-.101	.032	.03786
BA	.016	-.134	-.040	.04528
3 ³ M	-.393	.344	-.033	*
Repl.‡ Sed.	-.22	.15	-1.04	.4257
RW	-.119	-.037	.472	.07150
<i>Alluvial Soils</i>				
Six-tr. L	.514 _s	-.086		.05843
2 ³ K	.062	.092	-.108	.02543
C	-.103	.183	-.113	.11341
O	-.190	-.215	-.202	.04282

* Variances affected by adjustments.

H(u)	.07770	.08044	
H(b)	.07396	.08226	
A	.04044	.03967	.04019
M	.18266	.15849	.15043

s indicates $P < .05$ as determined for experiments individually.ss " $P < .01$ " " " "

§ Entries for the six-treatment series are NR = interaction of nitrogen with phosphate and potash jointly, with which is confounded a possible difference of nicifos v. sulphate of ammonia and super.

† Entries for the six-treatment series are PK_N, formally = PK in presence of nitrogen, but actually here represents difference of response to potash in presence of nicifos and of sulphate of ammonia. (See table 5 and associated discussion).

‡ Experiment N omitted owing to confounding of interactions with blocks.

Table D. *Interactions of Phosphate x Lime (CP), and responses to Lime (C) and to Nitrogen and Potash jointly (M).*

Soil	Expt.	CP	V(CP)	C	V(C)	M	V(M)
<i>Inland</i>	.. T	.064	.122	.327	.1133	-.809	.1981
	U	-.056	.078	-.111	.07166	-.565	.1265
	P	.054	.045	-.386	.04130	-.277	.0780
	BL	-.034	.070	.349	.06859	-.272	.1291
<i>Peat-clay</i>	.. V	—	—	.260	.10221	-.049	.1864

Table E. *Responses to Potash (K).*

Expt.		K	V (K)	K2 = (npk - np)	V (K2)	Tot. amt. of K ₂ O, oz. per tree
<i>Inland Soils.</i>						
Six-Tr.	E	-.027	.05804	.122	.1548	10.2
	D	.108	.04791	-.357	.1278	8.4
	G	-.919 _s	.17246	-1.112	.4599	11.0
	F	.042	.02827	-.171	.0753	7.2
	F31	-.065	.0879	.390	.2344	10.8 ?
	H	-.136	.03111	(u)-.232 (b) 1.453	.1457 .1489	7.2
	Average					9.1
2 ³	A	-.052	.04258	.346	.1587	3.53
	B	.270	.03786	.456	.1514	3.86
	BA	.070	.04528	.050	.1748	2.52
3 ³	M	-.114	.10029	.187	.1692†	4.0
						3.68
Repl.	N	-.450	.08050	-.154	.1431	?
	Sed.	-.250	.4257	-.957	1.7029	2.69 ?
	RW3	-.064	.07150	.370	.2860	2.69 ?
<i>Alluvial Soils.</i>						
Six-Tr.	L	-.462 _s	.04382	-.292	.1169	7.5
2 ³	K	-.602 _{ss}	.02534	-.345	.1888†	2.18
	C	-.278	.11341	-.472	.4536	3.19
3 ³	O	-.293	.02800	-.382	.0482‡	4.6 ?
						4.37

s: $P < .05$ ss: $P < .01$

† Confounded with blocks; error estimated from 3 d.f.

‡ Average of both levels of all ingredients: ignoring confounding of interactions with blocks.

Table F. *Responses to Nitrogen (N).*

		N	V(N)	N (12 units)	V(N ₁₂)
<i>Inland Soils.</i>					
Six-Tr.	E	.598s	.05804	.422	.02892
	D	.474s	.04791	.406	.03520
	G	.094	.17246	.061	.07335
	F	.159	.02827	.159	.02827
	F31	.55	.0879	.367	.03911
	H(u)	.201	.05532	.201	.05532
	H(b)	1.290ss	.05574	1.290	.05574
2 ³	A	.129	.04206	.147	.5494
	B	.240	.03786	.250	.04122
	BA	.130	.04528	.208	.11592
3 ³	M	-.362	.13252	-.362	.13252
Repl.	N	.286	.06440	.686	.37094
	Sed.	.71	.4257	1.065	.95782
	RW3	.108	.07150	.162	.16088
<i>Alluvial Soils.</i>					
Six-Tr.	L	-.245	.04382	-.235	.04038
2 ³	K	.202	.02534	.373	.08637
3 ³	O	.198	.028	.250	.04468
2 ³	C	-.390	.11341	-.390	.11341

s : $P < .05$ ss: $P < .001$ Table G. *Responses to Phosphate (P).*

		Expt.	P	Variance	Tot. P ₂ O ₅ oz. per tree
<i>Inland Soils.</i>					
Six-Tr.	E		1.112	.05804	8.40
	D		1.888	.04791	6.92
	H		2.142	.03035	5.93
	G		1.354	.17246	9.09
	F		.975	.02827	5.93
	F31		1.645	.0879	8.89
	A		.696	.03992	8.53
2 ³	B		1.335	.03786	9.34
	BA		2.124	.04528	6.09
	M		1.837	.11461	9.58
3 ³ Phos.	T		3.306	.20051	5.67
	U		1.723	.14099	8.19
	P		1.660	.07708	8.19
	BL		2.173	.12333	8.19
Repl.	N		2.347	.10733	6.50 ?
	Sed.		1.44	.4357	6.50 ?
	RW3		1.451	.07150	6.50 ?
<i>Alluvial Soils.</i>					
Six-Tr.	L		.434s	.04382	6.18
2 ³	K		.092	.02534	5.28
2 ³	C		-.010	.11341	7.71
3 ³	O		.124	.02800	5.49
Phos.	V		.384	.18960	5.67

s: $P = .05$.

All responses on inland soils are highly significant.

Table H. *Regressions on level of Phosphate.*

Expt.	P'	$b_{3.5}$	c	Variances x 100			p'
				$V(P')$	$V(b_{3.5})$	$V(c)$	
<i>Inland Soils.</i>							
3 ³							
M	.2296	.2612	-.03158	.1791	.2085	.02938	4.09
Phos.	.2395	.2370	-.03433	.1572	.1573	.05153	3.464
P	.2491	.2462	-.04074	.2888	.2877	.07542	3.464
U	.3373	.3105	-.04454	.2365	.2529	.04786	3.199
BL	.4833	.4722	-.02027	.3856	.4092	.07916	3.227
T*	—	.5910	-.04229	—	.30129	.34457	—
T†							
Repl.							
N		.3840	-.05352				
RW§		.1817	-.01791				
<i>Alluvial and peat-clay soils.</i>							
Phos.							
V	.0367	—	.02886	.3590	—	.07054	3.185
O	.0177†		0‡	.0571	—	—	—

* For 9 units of fertiliser as applied.

† Taking levels of p as 9/13ths of those in other experiments.

§ Weighted average from 2 adjacent experiments.

‡ Approximately. Some irregularities in data call for field inspection of soil conditions. For explanation of P', b, c and p', see text.



